University at Albany, SUNY

College of Engineering and Applied Sciences, Computer Science

ISCI/ISEN-210: Discrete Structures

Spring 2019

Final Exam

Name: ________________ ID #: ________________ Score: ________________

- This is a CLOSE BOOK & CLOSE NOTE exam. Also, you cannot access the Internet or use your laptop computer. Do the exam independently.
- There are a total of 100 points in the exam. Plan your work accordingly.
- Write out the steps for all problems to receive the full credit. Use additional pages if necessary.
- Date: May 13, 2019.
- Location: Lecture center 25.
- Time: 3:30 pm - 5:30 pm.

Problem	Points	Scores
Problem 1: True or False	20	
Problem 2: Logic and Proofs	10	
Problem 3: Set, Functions and Sequences	20	
Problem 4: Induction and Recursion	10	
Problem 5: Counting and Probability	15	
Problem 6: Relations	25	

Problem 1: True or False (20 points)

(1) The expression $\forall x \exists y \exists z P(x, y, z, c)$ is a well-formed formula.
○ True ○ False

(2) The time complexity of a recursive algorithm may depend critically on the number of recursive calls it makes.
○ True ○ False

(3) Let $f : Z \to Z$ be defined by $f(x) = 5x^3 - x$. Then the function $f(x)$ is an one-to-one (injective) and onto (surjective) function.
○ True ○ False

(4) Greedy algorithm can guarantee the smallest number of coins in the coin exchange optimization problem.
○ True ○ False

(5) The set $S = \{2n - 1 | n \text{ is a natural number}\}$ is countably infinite.
○ True ○ False

(6) Both the sender and the receiver share the same private key in the RSA cryptosystem.
○ True ○ False

(7) Let $\mathbf{A}$ and $\mathbf{B}$ are two arbitrary $n \times n$ matrix. $\mathbf{AB} = \mathbf{BA}$ always holds.
○ True ○ False

(8) Two random variables X and Y are said to be independent, if and only if $P(X, Y) = P(X)P(Y)$.
○ True ○ False

(9) If $a \equiv b (\text{mod } m)$ holds then $a \div c \equiv b \div c (\text{mod } m)$ holds, where c is any integer.
○ True ○ False

(10) The relation R on a set A is transitive if and only if $R^n \subseteq R$ for all positive integers n.
○ True ○ False

Problem 2: Logic and Proofs (10 points)

(1) [**3 points**] Describe a way to prove the contrapositive eqvalance $p \to q \equiv \neg q \to \neg p$.

(2) [**7 points**] Prove that if n is an integer and 3n + 2 is even, then n is even using a proof by contraposition.

Problem 3: Set, Functions and Sequences (20 points)

(1) [**5 points**] What is the power set of the set {0, 1, 3}?

(2) [**7 points**] Let f be the function from {a, b, c, d} to {1, 2, 3} defined by $f(a) = 3$, $f(b) = 2$, $f(c) = 1$, and $f(d) = 3$. Is f an one-to-one and onto function? Please give the necessary explanation to support your answer.

(3) [**8 points**] Calculate the summaztion $\sum_{i=1}^{50} 7 \times 2^i - i + 2$. Note that you can include 2^{50} in your final answer.

Problem 4: Induction and Recursion (10 points)

(1) **[6 points]** Use mathematical induction to show that $\frac{2}{3}+\frac{2}{9}+\frac{2}{27}+\ldots+\frac{2}{3^n} = 1-\frac{1}{3^n}$ whenever n is a positive integer.

(2) **[4 points]** Using your own words to explicitly interpret a set S which is a recursively defined by the following three rules:

- Rule 1: $5 \in S$.
- Rule 2: if $x \in S$, then $x + 5 \in S$.
- Rule 3: S contains nothing else.

Problem 5: Counting and Probability (15 points)

(1) [**5 points**] An elementary school has 500 students. Use pigeonhole principle to show that at least two of them were born on the same day of the year.

(2) [**5 points**] What is the probability that a positive integer selected at random from the set of positive integers not exceeding 100 is divisible by either 2 or 5?

(3) [**5 points**] Use Bayes rule to find $p(F|E)$ if $p(E|F) = 1/3$, $p(E|\bar{F}) = 1/4$, $p(F) = 2/5$, and $p(\bar{F}) = 3/5$, where E and F are events from a sample space S.

Problem 6: Relations (25 points)

(1) **[10 points]** Show that the relation $R = \{(a, b)|a \equiv b(\text{ mod } 7)\}$ is an equivalence relation on the set of integers.

(2) **[5 points]** What is the equivalence class of 13, *i.e.*, $[13]_R$, with respect to the equivalence relation R in (1)?

(3) **[10 points]** Let R_1 and R_2 be relations on a set $A = \{1, 2, 3, 4\}$ represented by the following directed graphs. Find the relations $R_1 \cap R_2$, $R_1 \cup R_2$, $R_1 \circ R_2$. [Hint: You can convert the represention of both R_1 and R_2 to zero-one matrices.]

(a) Relation R_1.　　(b) Relation R_2.

Figure 1: Two graphs.