University at Albany, SUNY

College of Engineering and Applied Sciences, Computer Science

ICEN/ICSI-210: Discrete Structures

Spring 2019

Homework Set 2

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Assigned Date: Feb 4, 2019 (Monday).
Due Date: Feb 11, 2018 (Monday), 11:59 PM.
Collaboration Policy. Homeworks will be done individually: each student must hand in their own answers. Use of partial or entire solutions obtained from others or online is strictly prohibited.
Late Policy. If urgent or unusual circumstances prohibit you from submitting a homework assignment in time, please e-mail the instructor explaining the situation to get exempt from late penalty. Otherwise, any late submissions without consent from the instructor will result in exponential penalty – late for one day loses 25%, two days loses 50%, and so on and so forth. **Those submissions $\geq$ 3 hours after the deadline will be considered as "late submission" with no exemption.**
Submission Format. Electronic submission as a PDF file to blackboard is mandatory.

- You can write your solution in Word and save it as a PDF file.
- You also can write it on any physical papers and scan them to a PDF file.
- If you don't have condition to scan, you still can take pictures by your smart phone and convert images to a PDF file by the online tool (https://imagetopdf.com).
- If you have multiple PDF files, please combine them to a PDF file by the online tool (https://www.pdfmerge.com) or (https://www.ilovepdf.com/merge_pdf).

Problem 1: Compound positions (15 points) Determine whether each of these compound propositions is satisfiable.

(a) $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg s)$

(b) $(\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg s) \wedge (\neg p \vee \neg r \vee \neg s) \wedge (p \vee q \vee \neg r) \wedge (p \vee \neg r \vee \neg s)$

(c) $(p \vee q \vee r) \wedge (p \vee \neg q \vee \neg s) \wedge (q \vee \neg r \vee s) \wedge (\neg p \vee r \vee s) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee s) \wedge (\neg p \vee \neg r \vee \neg s)$

Problem 2: Translate English to Logic (15 points) Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

(a) No one is perfect.
(b) Not everyone is perfect.
(c) All your friends are perfect.
(d) At least one of your friends is perfect.
(e) Everyone is your friend and is perfect.
(f) Not everybody is your friend or someone is not perfect.

Problem 3: Translate Logic to English (15 points) Let $C(x, y)$ mean that student x is enrolled in class y, where the domain for x consists of all students in your school and the domain for y consists of all classes being given at your school. Express each of these statements by a simple English sentence.

(a) $C(\text{Randy Goldberg}, \text{CS 252})$
(b) $\exists x C(x, \text{Math 695})$
(c) $\exists y C(\text{Carol Sitea}, y)$
(d) $\exists x (C(x, \text{Math 222}) \wedge C(x, \text{CS 252}))$
(e) $\exists x \exists y \forall z ((x \neq y) \wedge (C(x, z) \to C(y, z)))$
(f) $\exists x \exists y \forall z ((x \neq y) \wedge (C(x, z) \leftrightarrow C(y, z)))$

Problem 4: Rules of Inference (15 points) What rule of inference is used in each of these arguments?

(a) Kangaroos live inAustralia and are marsupials. Therefore, kangaroos are marsupials.

(b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.

(c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she canwork as a lifeguard. Therefore, Linda can work as a lifeguard.

(d) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.

(e) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

Problem 5: Application of Rules of Inference (10 points)

Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$, $\forall x(\neg Q(x) \vee S(x))$, $\forall x(R(x) \rightarrow \neg S(x))$, and $\exists x \neg P(x)$ are true, then $\exists x \neg R(x)$ is true.

Problem 6: Proofs (30 points)

(a) Use a proof by contraposition to show that if $x + y \geq 2$, where x and y are real numbers, then $x \geq 1$ or $y \geq 1$.

(b) Prove that if n is an integer and $3n + 2$ is even, then n is even using (1) a proof by contraposition, and (2) a proof by contradiction.

(c) Prove the triangle inequality, which states that if x and y are real numbers, then $|x| + |y| \geq |x + y|$ (where $|x|$ represents the absolute value of x, which equals x if $x \geq 0$ and equals $-x$ if $x < 0$).

[Optional Problem]: Proof Methods and Strategy (20 points)

(a) Prove that there are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 14$.

(b) Prove that $\sqrt[3]{2}$ is irrational.