University at Albany, SUNY

College of Engineering and Applied Sciences, Computer Science

ICEN/ICSI-210: Discrete Structures

Spring 2019

Homework Set 3

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Assigned Date: Feb 11, 2019 (Monday).
Due Date: Feb 18, 2018 (Monday), 11:59 PM.
Collaboration Policy. Homeworks will be done individually: each student must hand in their own answers. Use of partial or entire solutions obtained from others or online is strictly prohibited.
Late Policy. If urgent or unusual circumstances prohibit you from submitting a homework assignment in time, please e-mail the instructor explaining the situation to get exempt from late penalty. Otherwise, any late submissions without consent from the instructor will result in exponential penalty – late for one day loses 25%, two days loses 50%, and so on and so forth. **Those submissions $\geq$ 3 hours after the deadline will be considered as "late submission" with no exemption.**
Submission Format. Electronic submission as a PDF file to blackboard is mandatory.

- You can write your solution in Word and save it as a PDF file.
- You also can write it on any physical papers and scan them to a PDF file.
- If you don't have condition to scan, you still can take pictures by your smart phone and convert images to a PDF file by the online tool (https://imagetopdf.com).
- If you have multiple PDF files, please combine them to a PDF file by the online tool (https://www.pdfmerge.com) or (https://www.ilovepdf.com/merge_pdf).

Problem 1: Set (20 points) Let $A = \{a, b, c\}$, $B = \{x, y\}$, $C = \{x \in \mathbf{Z} | 0 \leq x \leq 1\}$, $D = \{x \in \mathbf{N} | x \leq 5\}$, and $E_i = \{1, 2, 3, \ldots, i\}$, find
(a) $A \times B \times C$
(b) $B \times A \times C$
(d) C^3
(e) $C \cup D$

(f) $C \cap D$
(g) $C - D$
(h) $D - C$
(i) $\bigcup_{i=1}^{n} E_i$
(j) $\bigcap_{i=1}^{n} E_i$

Problem 2: Set Equivalence (20 points) Suppose there are two sets C and D such that $C \subseteq D$.

(a) [3 points] Use a Venn diagram to illustrate the relationship $C \subseteq D$.

(b) [5 points] Use the law $A = B \equiv (A \subseteq B) \cap (B \subseteq A)$ to prove that $C \cup D = D$.

(c) [5 points] Use the law $A = B \equiv (A \subseteq B) \cap (B \subseteq A)$ to prove that $C \cap D = C$.

(d) [7 points] Let C indicates the positive multiples of 6, and D is the positive multiples of 3. Prove that $C \cup D = D$ and $C \cap D = C$.

Problem 3: Function with One-to-one Correspondence (40 points) Determine whether each of these functions is a bijection from **R** to **R**. If it is, then derive its inverse function.

(a) $f(x) = 2x + 1$
(b) $f(x) = x^2 + 1$
(c) $f(x) = x^3$
(d) $f(x) = (x^2 + 1)/(x^2 + 2)$

Problem 4: Function Composition (20 points).

(a) [5 points] Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from **R** to **R**.

(b) [5 points] Find $f + g$ and fg for the functions f and g given (a).

(c) [10 points] Let $f(x) = ax + b$ and $g(x) = cx + d$, where a, b, c, and d are constants. Determine necessary and sufficient conditions on the constants a, b, c, and d so that $f \circ g = g \circ f$.

[Optional Problem]: Floor and Ceiling Functions (20 points)

(a) Draw the graph of the function $f(x) = \lfloor x \rfloor + \lfloor x/2 \rfloor$ from **R** to **R**.

(b) Draw the graph of the function $f(x) = \lceil x \rceil + \lfloor x/2 \rfloor$ from **R** to **R**.