University at Albany, SUNY

College of Engineering and Applied Sciences, Computer Science

ICEN/ICSI-210: Discrete Structures

Spring 2019

Homework Set 8

Chengjiang Long

Assigned Date: Mar 25, 2019 (Monday).
Due Date: Apr 1, 2019 (Monday), 11:59 PM.
Collaboration Policy. Homeworks will be done individually: each student must hand in their own answers. Use of partial or entire solutions obtained from others or online is strictly prohibited.
Late Policy. If urgent or unusual circumstances prohibit you from submitting a homework assignment in time, please e-mail the instructor explaining the situation to get exempt from late penalty. Otherwise, any late submissions without consent from the instructor will result in exponential penalty – late for one day loses 25%, two days loses 50%, and so on and so forth. **Those submissions $\geq$ 3 hours after the deadline will be considered as "late submission" with no exemption.**
Submission Format. Electronic submission as a PDF file to blackboard is mandatory.

- You can write your solution in Word and save it as a PDF file.
- You also can write it on any physical papers and scan them to a PDF file.
- If you don't have condition to scan, you still can take pictures by your smart phone and convert images to a PDF file by the online tool (https://imagetopdf.com).
- If you have multiple PDF files, please combine them to a PDF file by the online tool (https://www.pdfmerge.com) or (https://www.ilovepdf.com/merge_pdf).

Problem 1: Mathematic Induction for Equality (20 points) Let $P(n)$ be the statement that $1^2 + 2^2 + \ldots + n^2 = n(n+1)(2n+1)/6$ for the positive integer n.
(a) [3 points] What is the statement $P(1)$?.
(b) [3 points] Show that $P(1)$ is true, completing the basis step of the proof.
(c) [3 points] What is the inductive hypothesis?

(d) [3 points] What do you need to prove in the inductive step?

(e) [4 points] Complete the inductive step, identifying where you use the inductive hypothesis.

(f) [4 points] Explain why these steps show that this formula is true whenever n is a positive integer.

Problem 2: Mathematic Induction for Inequality and Property (30 points)

(1) Use mathematic induction to prove that $2^n > n^2$ if n is an integer greater than 4.

(2) Use mathematic induction to prove that $n^2 - 7n + 12$ is nonnegative whenever n is an integer with $n \geq 3$.

Problem 3: Strong Induction (30 points)

Let $P(n)$ be the statement that a postage of n cents can be formed using just 4-cent stamps and 7-cent stamps. The parts of this exercise outline a strong induction proof that $P(n)$ is true for $n \geq 18$.

(1) [4 points] Show statements $P(18)$, $P(19)$, $P(20)$, and $P(21)$ are true, completing the basis step of the proof.

(2) [6 points] What is the inductive hypothesis of the proof?

(3) [6 points] What do you need to prove in the inductive step?

(4) [7 points] Complete the inductive step for $k \geq 21$.

(5) [7 points] Explain why these steps show that this statement is true whenever $n \geq 18$.

Problem 4: Well-Ordering Property (20 points)

Use the well-ordering property to prove the division algorithm. Recall that the division algorithm states that if a is an integer and d is a positive integer, then there are unique integers q and r with $0 \leq r < d$ and $a = dq + r$.

[Optional] Extra Points (20 points)

(a) [10 points] Use the well-ordering principle to prove the Archimedian property which states that if a and b are positive integers, then there exists some positive integer n such that $na \geq b$.

(b) [10 points] Use the well-ordering principle to show that if x and y are real numbers with $x < y$, then there is a rational number r with $x < r < y$.

[**Hint**: *Use the Archimedean property to find a positive integer A with $A > 1/(y - x)$. Then show that there is a rational number r with denominator A between x and y by looking at the numbers $\lfloor x \rfloor + j/A$, where j is a positive integer.*]