University at Albany, SUNY

College of Engineering and Applied Sciences, Computer Science

ISEN/ISCI-210: Discrete Structures

Spring 2019

Midterm Exam 1

Name: ______________ ID #: ______________ Score: ______________

- This is a CLOSE BOOK & CLOSE NOTE exam. Also, you cannot access the Internet or use your laptop computer. Do the exam independently.
- There are a total of 100 points in the exam. Plan your work accordingly.
- Write out the steps for all problems to receive the full credit. Use additional pages if necessary.
- Date: March 8th, 2019.
- Location: Lecture center hall 25.
- Time: 11:30 am - 12:25 pm.

Problem	Points	Scores
Problem 1: Logical Equivalence	20	
Problem 2: Sequence and Summation	30	
Problem 3: Functions	30	
Problem 4: Algorithm, Growth Function and Complexity	20	

Logical Equivalence Laws.

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \to q \equiv \neg p \vee q$
$p \to q \equiv \neg q \to \neg p$
$p \vee q \equiv \neg p \to q$
$p \wedge q \equiv \neg(p \to \neg q)$
$\neg(p \to q) \equiv p \wedge \neg q$
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

TABLE 8 Logical Equivalences Involving Biconditional Statements.

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_n$$

$$S_n = \frac{n(a_1 + a_n)}{2}$$

$$S_n = \frac{n(a_1 + a_1 + (n-1)d)}{2}$$

$$S_n = \begin{cases} \frac{a_1(1-r^n)}{1-r} & r \neq 1 \\ na_1 & \mathrm{r} = 1 \end{cases}$$

$$S_n = \frac{n(2a_1 + (n-1)d)}{2}$$

Equations for Summation Calculation in Sequences.

Problem 1: Logical Equivalence (20 points)

Show that $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$ are logically equivalent.

(1) [**8 points**] By constructing truth table.

(2) [**12 points**] By applying the known logical equivalence laws provided on the 2nd page.

Problem 2: Sequence and Summation (30 points)

What is the value of each of these sums: *(Hint: you can use the equations provided on the 2nd page.)*

(1) **[10 points]** $\sum_{i=1}^{50}(3i+1)$.

(2) **[10 points]** $\sum_{i=1}^{50} 3 \times 2^i$. *(The result can be an expression with* 2^{50}*)*

(3) **[10 points]** $\sum_{i=1}^{50} \frac{1}{i(i+1)}$.

Problem 3: Functions (30 points)

Let f be a function from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = 3x + 1$.

(1) **[12 points]** Prove that $f(x)$ is an one-to-one correspondence (bijection) function.

(2) **[6 points]** Derive the inverse function for $f(x)$.

(3) **[12 points]** Let $g(x) = x^2$ from $\mathbb{R}$ to $\mathbb{R}$. What are the composition functions $f \circ g$ and $g \circ f$?

Problem 4: Algorithm, Growth Function and Complexity (20 points)

(1) [**8 points**] Define the statement $f(x)$ is $O(g(x))$.

(2) [**12 points**] Show that $(logn + n^2)^2$ is $O(n^4)$.