University at Albany, SUNY

College of Engineering and Applied Sciences, Computer Science

ISEN/ISCI-210: Discrete Structures

Spring 2019

Midterm Exam 2

Name: ________________ ID #: ________________ Score: ________________

- This is a CLOSE BOOK & CLOSE NOTE exam. Also, you cannot access the Internet or use your laptop computer. Do the exam independently.
- There are a total of 100 points in the exam. Plan your work accordingly.
- Write out the steps for all problems to receive the full credit. Use additional pages if necessary.
- Date: April 17th, 2019.
- Location: Lecture center hall 25.
- Time: 11:30 am - 12:25 pm.

Problem	Points	Scores
Problem 1: Greatest Common Divisor and Least Common Multiple	25	
Problem 2: Integer Representations and Modular Exponentiation	20	
Problem 3: Induction and Recursion	20	
Problem 4: Counting	35	

Problem 1: Greatest Common Divisor and Least Common Multiple (25 points)

(1) [**10 points**] What are the greatest common divisor and the least common multiple for $3^7 \cdot 5^3 \cdot 7^3$ and $2^{11} \cdot 3^5 \cdot 5^9$?

(2) [**15 points**] Calculate the greatest common divisor $gcd(125, 75)$ between 125 and 75 using Euclidean algorithm?

Problem 2: Integer Representations and Modular Exponentiation (20 points)

(1) [**5 points**] Convert 23 to binary, octal and hexadecimal representations.

(2) [**15 points**] Calculate Modular Exponention 11^{23} mod 9.

Problem 3: Induction and Recursion (20 points)

(1) [**10 points**] Use mathematical induction to show that

$$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \ldots + \frac{1}{(2n-1) \times (2n+1)} = \frac{n}{2n+1} \tag{1}$$

whenever n is a positive integer.

(2) [**10 points**] Give a recursive definition of the set of positive multiples of 5.

Problem 4: Counting (35 points)

(1) **[10 points]** Use the pegionhole principle to show that if there are 30 students in a class, then at least two have last names that begin with the same letter.

(2) **[10 points]** We say a bit string is unbalanced if there are more ones than zeroes or more zeros than ones. How many 10-bit strings are unbalanced?

(3) **[15 points]** Let n be a positive integer. Use the binomial theorem, *i.e.*, $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$, to prove that

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$$

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