University at Albany, SUNY

College of Engineering and Applied Sciences, Computer Science

ISEN/ISCI-210: Discrete Structures

Fall 2018

Homework Set 1

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Assigned Date: Sep 7, 2018 (Friday).
Due Date: Sep 17, 2018 (Monday).
Collaboration Policy. Homeworks will be done individually: each student must hand in their own answers. Use of partial or entire solutions obtained from others or online is strictly prohibited.
Late Policy. No late submissions will be allowed without consent from the instructor. If urgent or unusual circumstances prohibit you from submitting a homework assignment in time, please e-mail me explaining the situation.
Submission Format. Electronic submission of a PDF file is mandatory.

Problem 1: Propositions (10 points) Let p, q, and r be the propositions

p : Grizzly bears have been seen in the area.

q : Hiking is safe on the trail.

r : Berries are ripe along the trail.

Write these propositions using p, q, and r and logical connectives (including negations).

(a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.

(b) Grizzly bears have not been seen in the area and hik- ing on the trail is safe, but berries are ripe along the trail.

(c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

(d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.

(e) For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.

(f) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

Problem 2: True tables (30 points) Construct a truth table for each of these compound propositions

(a) $(p \vee \neg q) \to q$
(b) $(p \to \neg q) \leftrightarrow (\neg q \to \neg p)$
(c) $(p \leftrightarrow q) \oplus (p \to \neg q)$
(d) $(p \vee q) \oplus (p \wedge q)$
(e) $(p \wedge q) \vee \neg r$
(f) $(p \to q) \wedge (\neg p \to r)$
(g) $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$
(h) $(p \to q) \oplus (\neg q \to r)$
(i) $((p \to q) \to r) \to s$
(j) $(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$

Problem 3: Equivalences (20 points).

(a) Use a truth table to verify the first De Morgan law: $\neg(p \wedge q) \equiv \neg p \vee \neg q$.

(b) Use De Morgan's laws to find the negation of each of the following statements: (1) Jan is rich and happy. (2) Carlos will bicycle or run tomorrow. (3) James is young and strong. (4) Rita will move to Oregon or Washington.

(c) Determine whether $(\neg q \wedge (p \to q)) \to \neg p$ is a tautology.

(d) Show that $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent.

(e) Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.

(f) Show that $(p \wedge q) \to r$ and $(p \to r) \wedge (q \to r)$ are not logically equivalent.

Problem 4: Compound positions (15 points) Determine whether each of these compound propositions is satisfiable.

(a) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$
(b) $(p \to q) \wedge (p \to \neg q) \wedge (\neg p \to q) \wedge (\neg p \to \neg q)$
(c) $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$

Problem 5: Predicatives and Quantifiers (10 points) Let $Q(x, y)$ be the statement "$x + y = x - y$." If the domain for both variables consists of all integers, what are the truth values?

(a) $Q(1, 1)$
(b) $Q(2, 0)$
(c) $\forall y Q(1, y)$
(d) $\exists x Q(x, 2)$
(e) $\exists x \exists y Q(x, y)$
(f) $\forall x \exists y Q(x, y)$
(g) $\exists y \forall x Q(x, y)$
(h) $\forall y \exists x Q(x, y)$
(i) $\forall x \forall y Q(x, y)$

Problem 6: Proofs (15 points)

(a) Use a proof by contraposition to show that if $x + y \geq 2$, where x and y are real numbers, then $x \geq 1$ or $y \geq 1$.

(b) Prove that if n is an integer and $3n + 2$ is even, then n is even using (1) a proof by contraposition, and (2) a proof by contradiction.

(c) Prove the triangle inequality, which states that if x and y are real numbers, then $|x| + |y| \geq |x + y|$ (where $|x|$ represents the absolute value of x, which equals x if $x \geq 0$ and equals $-x$ if $x < 0$).