University at Albany, SUNY

College of Engineering and Applied Sciences, Computer Science

ISEN/ISCI-210: Discrete Structures

Fall 2018

Midterm Exam 1

Name: ________________ ID #: ________________ Score: ________________

- This is a CLOSE BOOK & CLOSE NOTE exam. Also, you cannot access the Internet or use your laptop computer. Do the exam independently.
- Logical equivalence tables are given on Page 2.
- There are a total of 100 points in the exam. Plan your work accordingly.
- Write out the steps for all problems to receive the full credit. Use additional pages if necessary.
- Date: Oct 8th, 2018.
- Location: Lecture center hall 25.
- Time: 9:20 am - 10:20 am (can be extended to 10:35 am).

Problem	Points	Scores
Problem 1: True or False	20	
Problem 2: True Table and Logical Equivalence	20	
Problem 3: Predicatives and Quantifiers	20	
Problem 4: Set, Sequences and Summation	20	
Problem 5: Functions	20	

Figure 1: Logical equivalences.

Equivalence	***Name***
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

$p \rightarrow q \equiv \neg p \vee q$
$p \rightarrow q \equiv \neg q \rightarrow \neg p$
$p \vee q \equiv \neg p \rightarrow q$
$p \wedge q \equiv \neg(p \rightarrow \neg q)$
$\neg(p \rightarrow q) \equiv p \wedge \neg q$
$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

(a) Involving conditional statements.

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

(b) Involving biconditional statements.

Figure 2: Logical equivalences involving conditional and biconditional statements.

Problem 1: True or False (20 points)

(1) The sentences "x + 2 = 11." and "Do not pass go." are propositions.
○ True ○ False

(2) $1 + 1 = 3$ if and only if monkeys can fly.
○ True ○ False

(3) For integer variable m and n, the true value of $\exists n \forall m (nm = m)$ is True.
○ True ○ False

(4) $\emptyset$ is an element in the set $\{\emptyset\}$.
○ True ○ False

(5) $\emptyset$ is a subset in the set $\{\emptyset\}$.
○ True ○ False

(6) If A and B are sets, then $A - B$ is equivalent to $B - A$.
○ True ○ False

(7) The function $f(x) = 2x - 3$ is a one-to-one (injective) and onto (surjective) function from interger numbers $\mathbf{Z}$ to $\mathbf{Z}$.
○ True ○ False

(8) The function $f(x) = 2x - 3$ is a one-to-one (injective) and onto (surjective) function from natural numbers $\mathbf{N}$ to $\mathbf{N}$.
○ True ○ False

(9) The set $C = \{2n - 1 \mid n \text{ is a natural number}\}$ (the odd natural numbers) is countably infinite.
○ True ○ False

(10) Let $\mathbf{A}$ and $\mathbf{B}$ are two arbitrary 2×2 matrix. $\mathbf{AB} = \mathbf{BA}$.
○ True ○ False

Problem 2: True Table and Logical Equivalence (20 points)

(1) [8 points] Use a truth table to verify the first De Morgan law $\neg(p \wedge q) \equiv \neg p \vee \neg q$.

(2) [12 points] Show that $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent.

Problem 3: Predicatives and Quantifiers (20 points)

Let $Q(x)$ be the statement "$x+1 > 2x$." If the domain consists of all integers, what are these truth values for the following statements and why? Explain your solution with details.

(1) $Q(0)$, $Q(-1)$, and $Q(1)$.
(2) $\exists x Q(x)$
(3) $\forall x Q(x)$
(4) $\exists x \neg Q(x)$
(5) $\forall x \neg Q(x)$.

Problem 4: Set, Sequences and Summation (20 points)

(1) [**4 points**] Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$. Find $A \vee B$, $A \wedge B$, $A - B$ and $B - A$.

(2) [**6 points**] For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list. Assuming that your formula or rule is correct, determine the next three terms of the sequence.

[a] $15, 8, 1, -6, -13, -20, -27, ...$

[b] $3, 6, 12, 24, 48, 96, 192, ...$

(3) [**10 points**] Compute the sum $\sum_{j=1}^{20}(1 + 3j + (-1)^j)$.

Problem 5: functions (20 points)

(a) [**10 points**] Show the function $f(x) = x^3 + 1$ is a bijection from **R** to **R**.

(b) [**10 points**] Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from **R** to **R**.