

University at Albany, SUNY

College of Engineering and Applied Sciences, Computer Science

ICSI-521: Discrete Mathematics with Applications Spring 2019

Homework Set 1

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Assigned Date: Jan 31, 2019 (Thursday).

Due Date: Feb 14, 2018 (Thursday), 11:59 PM.

Collaboration Policy. Homeworks will be done individually: each student must hand in their own answers. Use of partial or entire solutions obtained from others or online is strictly prohibited.

Late Policy. If urgent or unusual circumstances prohibit you from submitting a homework assignment in time, please e-mail the instructor explaining the situation to get exempt from late penalty. Otherwise, any late submissions without consent from the instructor will result in exponential penalty – late for one day loses 25%, two days loses 50%, and so on and so forth. **Those submissions ≥ 3 hours after the deadline will be considered as “late submission” with no exemption.**

Submission Format. Electronic submission as a zip file including a PDF file and code files to blackboard is mandatory.

- You can write your solution in Word and save it as a PDF file.
- You also can write it on any physical papers and scan them to a PDF file.
- If you don't have condition to scan, you still can take pictures by your smart phone and convert images to a PDF file by the online tool (<https://imagetopdf.com>).
- If you have multiple PDF files, please combine them to a PDF file by the online tool (<https://www.pdfmerge.com>) or (https://www.ilovepdf.com/merge_pdf).

Problem 1: Equivalences (20 points).

- Determine whether $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology.
- Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent.
- Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.

(d) Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent.

Problem 2: Compound positions (15 points) Determine whether each of these compound propositions is satisfiable.

- (a) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$
- (b) $(p \rightarrow q) \wedge (p \rightarrow \neg q) \wedge (\neg p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$
- (c) $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$

Problem 3: Proofs (10 points)

(a) Prove that if n is an integer and $3n + 2$ is even, then n is even using (1) a proof by contraposition, and (2) a proof by contradiction.

(b) Prove the triangle inequality, which states that if x and y are real numbers, then $|x| + |y| \geq |x + y|$ (where $|x|$ represents the absolute value of x , which equals x if $x \geq 0$ and equals $-x$ if $x < 0$).

Problem 4: Mathematic Induction and Strong Induction (15 points)

(a) Let $P(n)$ be the statement that $1^2 + 2^2 + \dots + n^2 = n(n + 1)(2n + 1)/6$ for the positive integer n .

(b) Redefine $P(n)$ to prove that $2^n > n^2$ if n is an integer greater than 4.

(c) The Fibonacci sequence is defined by $F_{n+2} = F_{n+1} + F_n$ for $n \geq 1$ with starting values $F_1 = F_2 = 1$. Show that

$$F_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1 + \sqrt{5}}{2} \right)^n - \left(\frac{1 - \sqrt{5}}{2} \right)^n \right] \quad (1)$$

Problem 5: Recursive Definitions and Structural Induction (20 points)

Let S be the subset of the set of ordered pairs of integers defined recursively by

Basis step: $(0, 0) \in S$.

Recursive step: If $(a, b) \in S$, then $(a + 2, b + 3) \in S$ and $(a + 3, b + 2) \in S$.

(a) [5 points] List the elements of S produced by the first five applications of the recursive definition.

(b) [10 points] Show that $5|a + b$ when $(a, b) \in S$ [**Hint:** you can use strong induction or structural induction].

Problem 6: Computer Projects (20 points)

Write a program with any programming language you like to solve and answer the following problems. Please keep in mind that you should provide details

to run your program and testing cases with necessary descriptions to make a solid solution.

(a) Given a nonnegative integer n , find the n -th Fibonacci number directly using the following closed-form expression:

$$F_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1 + \sqrt{5}}{2} \right)^n - \left(\frac{1 - \sqrt{5}}{2} \right)^n \right] \quad (2)$$

(b) Given a nonnegative integer n , find the n -th Fibonacci number using iteration.

(c) Given a nonnegative integer n , find the n -th Fibonacci number using recursion.

(d) Compare either the number of operations or the time needed to compute Fibonacci numbers by the above three implementations.

[Optional Problem]: Well-Ordering Property (20 points)

Prove the Archimedian property which states that if a and b are positive integers, then there exists some positive integer n such that $na \geq b$.