

University at Albany, SUNY

College of Engineering and Applied Sciences, Computer Science

ICSI-521: Discrete Mathematics with Applications Spring 2019

Homework Set 2

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Assigned Date: Feb 14, 2019 (Thursday).

Due Date: Feb 28, 2018 (Thursday), 11:59 PM.

Collaboration Policy. Homeworks will be done individually: each student must hand in their own answers. Use of partial or entire solutions obtained from others or online is strictly prohibited.

Late Policy. If urgent or unusual circumstances prohibit you from submitting a homework assignment in time, please e-mail the instructor explaining the situation to get exempt from late penalty. Otherwise, any late submissions without consent from the instructor will result in exponential penalty – late for one day loses 25%, two days loses 50%, and so on and so forth. **Those submissions ≥ 3 hours after the deadline will be considered as “late submission” with no exemption.**

Submission Format. Electronic submission as a zip file including a PDF file and code files to blackboard is mandatory.

- You can write your solution in Word and save it as a PDF file.
- You also can write it on any physical papers and scan them to a PDF file.
- If you don't have condition to scan, you still can take pictures by your smart phone and convert images to a PDF file by the online tool (<https://imagetopdf.com>).
- If you have multiple PDF files, please combine them to a PDF file by the online tool (<https://www.pdfmerge.com>) or (https://www.ilovepdf.com/merge_pdf).

Problem 1: Set Equivalence (20 points) Suppose there are two sets C and D such that $C \subseteq D$.

- (a) [3 points] Use a Venn diagram to illustrate the relationship $C \subseteq D$.

(b) [5 points] Use the law $A = B \equiv (A \subseteq B) \cap (B \subseteq A)$ to prove that $C \cup D = D$.

(c) [5 points] Use the law $A = B \equiv (A \subseteq B) \cap (B \subseteq A)$ to prove that $C \cap D = C$.

(d) [7 points] Let C indicate the positive multiples of 6, and D is the positive multiples of 3. Prove that $C \cup D = D$ and $C \cap D = C$.

Problem 2: Function Composition (15 points).

(a) Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from \mathbf{R} to \mathbf{R} .

(b) Find $f + g$ and fg for the functions f and g given (a).

(c) Let $f(x) = ax + b$ and $g(x) = cx + d$, where a, b, c , and d are constants. Determine necessary and sufficient conditions on the constants a, b, c , and d so that $f \circ g = g \circ f$.

Problem 3: Cardinality of Sets (5 points) Determine whether each of these sets is finite, countably infinite, or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.

- (a) the odd negative integers.
- (b) the integers with absolute value less than 1,000,000.
- (c) the real numbers between 0 and 2.
- (d) the set $A \times \mathbf{Z}^+$ where $A = 2, 3$.
- (e) the integers that are multiples of 10.

Problem 4: Zero-one Matrices (10 points)

(a) [3 points] Find the Boolean product of \mathbf{A} and \mathbf{B} , where

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

(d) [7 points] Find $\mathbf{A}^{[2]}$, $\mathbf{A}^{[3]}$ and $\mathbf{A} \vee \mathbf{A}^{[2]} \vee \mathbf{A}^{[3]}$ with \mathbf{A} given in (a).

Problem 5: Growth of Functions and Complexity (15 points)

(a) Show that $(x^2 + xy + xlogy)^3$ is $O(x^6y^3)$.

(b) Show that $x^5y^3 + x^4y^4 + x^3y^5$ is $\Omega(x^3y^3)$.

(c) Show that $3x^2 + x + 1$ is $\Theta(3x^2)$.

(d) Suppose that $f(x)$, $g(x)$, and $h(x)$ are functions such that $f(x)$ is $\Theta(g(x))$ and $g(x)$ is $\Theta(h(x))$. Show that $f(x)$ is $\Theta(h(x))$.

(e) Show that if $f_1(x)$ and $f_2(x)$ are functions from the set of positive integers to the set of real numbers and $f_1(x)$ is $\Theta(g_1(x))$ and $f_2(x)$ is $\Theta(g_2(x))$, then

$(f_1 f_2)(x)$ is $\Theta((g_1 g_2)(x))$.

Problem 6: Computer Projects (35 points)

Write a program with any programming language you like to solve and answer the following problems. Please keep in mind that you should provide details to run your program and testing cases with necessary descriptions, as well as the complexity analysis, to make a solid solution.

Say you have an array for which the i -th element is the price of a given stock on day i .

(a) [5 points] If you were only permitted to complete at most one transaction (ie, buy one and sell one share of the stock), design an algorithm to find the maximum profit.

Example 1

Input: [7,1,5,3,6,4]

Output: 5

Explanation: Buy on day 2 (price = 1) and sell on day 5 (price = 6), profit = 6-1 = 5. Not 7-1 = 6, as selling price needs to be larger than buying price.

Example 2

Input: [7,6,4,3,1]

Output: 0

Explanation: In this case, no transaction is done, *i.e.*, max profit = 0.

(b) [10 points] Design an algorithm to find the maximum profit. You may complete as many transactions as you like (*i.e.*, buy one and sell one share of the stock multiple times).

Note: You may not engage in multiple transactions at the same time (*i.e.*, you must sell the stock before you buy again).

Example 1

Input: [7,1,5,3,6,4]

Output: 7

Explanation: Buy on day 2 (price = 1) and sell on day 3 (price = 5), profit = 5-1 = 4. Then buy on day 4 (price = 3) and sell on day 5 (price = 6), profit = 6-3 = 3.

Example 2

Input: [1,2,3,4,5]

Output: 4

Explanation: Buy on day 1 (price = 1) and sell on day 5 (price = 5), profit = 5-1 = 4. Note that you cannot buy on day 1, buy on day 2 and sell them later, as you are engaging multiple transactions at the same time. You must sell before buying again.

Example 3

Input: [7,6,4,3,1]

Output: 0

Explanation: In this case, no transaction is done, *i.e.*, max profit = 0.

(c) [10 points] Design an algorithm to find the maximum profit. You may complete at most two transactions.

Note: You may not engage in multiple transactions at the same time (*i.e.*, you must sell the stock before you buy again).

Example 1

Input: [3,3,5,0,0,3,1,4]

Output: 6

Explanation: Buy on day 4 (price = 0) and sell on day 6 (price = 3), profit = $3-0 = 3$. Then buy on day 7 (price = 1) and sell on day 8 (price = 4), profit = $4-1 = 3$.**Example 2**

Input: [1,2,3,4,5]

Output: 4

Explanation: Buy on day 1 (price = 1) and sell on day 5 (price = 5), profit = $5-1 = 4$. Note that you cannot buy on day 1, buy on day 2 and sell them later, as you are engaging multiple transactions at the same time. You must sell before buying again.**Example 3**

Input: [7,6,4,3,1]

Output: 0

Explanation: In this case, no transaction is done, *i.e.*, max profit = 0.

(d) [10 points] Design an algorithm to find the maximum profit. You may complete at most k transactions.

Note: You may not engage in multiple transactions at the same time (*i.e.*, you must sell the stock before you buy again).

Example 1

Input: [2,4,1], k = 2

Output: 2

Explanation: Buy on day 1 (price = 2) and sell on day 2 (price = 4), profit = $4-2 = 2$.

Example 2

Input: [3,2,6,5,0,3], $k = 2$

Output: 7

Explanation: Buy on day 2 (price = 2) and sell on day 3 (price = 6), profit = $6 - 2 = 4$. Then buy on day 5 (price = 0) and sell on day 6 (price = 3), profit = $3 - 0 = 3$.

[Optional Problem] (20 points)

- (a) Show that the union of a countable number of countable sets is countable.
- (b) Show that the set of all finite bit strings is countable. [**Hint:** *you can view (b) as a special case of (a).*]