# University at Albany, SUNY

College of Engineering and Applied Sciences, Computer Science

# ICSI-521: Discrete Mathematics with Applications

Spring 2019

## Homework Set 4

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**Assigned Date**: Mar 14, 2019 (Thursday).
**Due Date**: Mar 28, 2018 (Thursday), 11:59 PM.
**Collaboration Policy**. Homeworks will be done individually: each student must hand in their own answers. Use of partial or entire solutions obtained from others or online is strictly prohibited.
**Late Policy**. If urgent or unusual circumstances prohibit you from submitting a homework assignment in time, please e-mail the instructor explaining the situation to get exempt from late penalty. Otherwise, any late submissions without consent from the instructor will result in exponential penalty – late for one day loses 25%, two days loses 50%, and so on and so forth. **Those submissions $\geq$ 3 hours after the deadline will be considered as "late submission" with no exemption.**
**Submission Format**. Electronic submission as a zip file including a PDF file and code files to blackboard is mandatory.

- You can write your solution in Word and save it as a PDF file.
- You also can write it on any physical papers and scan them to a PDF file.
- If you don't have condition to scan, you still can take pictures by your smart phone and convert images to a PDF file by the online tool (https://imagetopdf.com).
- If you have multiple PDF files, please combine them to a PDF file by the online tool (https://www.pdfmerge.com) or (https://www.ilovepdf.com/merge_pdf).

**Problem 1: Pigeonhole Principle (15 points)** Use the pigeonhole principle to prove the following statements.

(a) Show that if there are 30 students in a class, then at least two have last names that begin with the same letter

(b) Suppose that every student in a discrete mathematics class of 25 students is a freshman, a sophomore, or a junior. Show that there are at least nine freshmen, at least nine sophomores, or at least nine juniors in the class.

(c) What is the minimum number of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee that there are at least 100 who come from the same state?

**Problem 2: Permutations and Combinations (25 points)**

(a) Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have the same number of men and women?

(b) Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have more women than men?

(c) How many bit strings contain exactly eight 0s and ten 1s if every 0 must be immediately followed by a 1?

(d) How many bit strings contain exactly five 0s and fourteen 1s if every 0 must be immediately followed by two 1s?

(e) How many bit strings of length 10 contain at least three 1s and at least three 0s?

**Problem 3: Binomial Coefficients and Identities (10 points)**

(a) [3 points] Find the expansion of $(x+y)^5$ using using the binomial theorem.

(b) [3 points] What is the coefficient of $x^9$ in $(2-x)^{19}$?

(c) [4 points] The row of Pascal's triangle containing the binomial coefficients $\binom{10}{k}$, $0 \le k \le 10$, is:

1, 10, 45, 120, 210, 252, 210, 120, 45, 10, 1

Use Pascal's identity to produce the row immediately following this row in Pascal's triangle.

**Problem 4: r-Perputaiton and r-Combination (10 points)**

(a) [5 points] Find the coefficient of $x^3y^2z^5$in $(x+y+z)^{10}$.

(b) [5 points] How many terms are there in the expansion of $(x+y+z)^{100}$?

**Problem 5: Computer Projects (40 points)**

*Write a program with any programming language you like to solve and answer the following problems. Please keep in mind that you should provide details to run your program and testing cases with necessary descriptions, as well as the complexity analysis, to make a solid solution.*

(a) [10 points] The set $[1, 2, 3, ..., n]$ contains a total of $n!$ unique permutations. By listing and labeling all of the permutations in order, we get the following sequence for $n = 3$:
"123"
"132"
"213"
"231"
"312"
"321"
Given $n$ and $k$, return the $k$-th permutation sequence.

**Note**: (1) Given $n$ will be between 1 and 9 inclusive. (2) Given $k$ will be between 1 and $n!$ inclusive.

**Example 1**
Input: $n = 3$, $k = 3$
Output: "213"

**Example 2**
Input: n = 4, k = 9
Output: "2314"

(b) [10 points] Given a set of candidate numbers (candidates) (without duplicates) and a target number (target), find all unique combinations in candidates where the candidate numbers sums to target. The same repeated number may be chosen from candidates unlimited number of times.

**Note**: (1) All numbers (including target) will be positive integers. (2) The solution set must not contain duplicate combinations.

**Example 1**
Input: candidates = [2,3,6,7], target = 7,
A solution set is:
[
[7],
[2,2,3]
]

**Example 2**
Input: candidates = [2,3,5], target = 8,
A solution set is:
[
[2,2,2,2],
[2,3,3],
[3,5]
]

(c) [10 points] Given a collection of candidate numbers (candidates) and a target number (target), find all unique combinations in candidates where the candidate numbers sums to target. Each number in candidates may only be used once in the combination.

**Note**: (1) All numbers (including target) will be positive integers. (2) The solution set must not contain duplicate combinations.

**Example 1**
Input: candidates = [10,1,2,7,6,1,5], target = 8,
A solution set is:
[
[1, 7],
[1, 2, 5],
[2, 6],
[1, 1, 6]
]

**Example 2**
Input: candidates = [2,5,2,1,2], target = 5,
A solution set is:
[
[1,2,2],
[5]
]

(d) [10 points] Find all possible combinations of $k$ numbers that add up to a number $n$, given that only numbers from 1 to 9 can be used and each combination should be a unique set of numbers.

**Note**: (1) All numbers will be positive integers. (2) The solution set must not contain duplicate combinations.

**Example 1**
Input: $k = 3$, $n = 7$
Output: [[1,2,4]]

**Example 2**
Input: $k = 3$, $n = 9$
Output: [[1,2,6], [1,3,5], [2,3,4]]

**[Optional Problem] (20 points)**

(a) [5 points] Suppose that $k$ and $n$ are integers with $1 \leq k < n$. Prove the **hexagon identity**

$$\binom{n-1}{k-1}\binom{n}{k+1}\binom{n+1}{k} = \binom{n-1}{k}\binom{n}{k-1}\binom{n+1}{k+1} \tag{1}$$

which relates terms in Pascal's triangle that form a hexagon.

A **circular $r$-permutation** of $n$ people is a seating of $r$ of thesen people around a circular table, where seatings are considered to be the same if they can be obtained from each other by rotating the table.

(b) [3 points] Find the number of circular 3-permutations of 5 people.

(c) [6 points] Find a formula for the number of circular $r$-permutations of $n$ people.

(d) [6 points] Find a formula for the number of ways to seatr of $n$ people around a circular table, where seatings are considered the same if every person has the same two neighbors without regard to which side these neighbors are sitting on.