University at Albany, SUNY

College of Engineering and Applied Sciences, Computer Science

ICEN/ICSI-210: Discrete Structures

Spring 2019

Homework Set 1

Chengjiang Long

Assigned Date: Jan 28, 2019 (Monday).
Due Date: Feb 4, 2018 (Monday), 11:59 PM.
Collaboration Policy. Homeworks will be done individually: each student must hand in their own answers. Use of partial or entire solutions obtained from others or online is strictly prohibited.
Late Policy. If urgent or unusual circumstances prohibit you from submitting a homework assignment in time, please e-mail the instructor explaining the situation to get exempt from late penalty. Otherwise, any late submissions without consent from the instructor will result in exponential penalty – late for one day loses 25%, two days loses 50%, and so on and so forth. **Those submissions $\geq$ 3 hours after the deadline will be considered as "late submission" with no exemption.**
Submission Format. Electronic submission as a PDF file to blackboard is mandatory.

- You can write your solution in Word and save it as a PDF file.
- You also can write it on any physical papers and scan them to a PDF file.
- If you don't have condition to scan, you still can take pictures by your smart phone and convert images to a PDF file by the online tool (https://imagetopdf.com).
- If you have multiple PDF files, please combine them to a PDF file by the online tool (https://www.pdfmerge.com) or (https://www.ilovepdf.com/merge_pdf).

Problem 1: True tables (30 points) Construct a truth table for each of these compound propositions

(a) $(p \vee \neg q) \rightarrow q$
(b) $(p \leftrightarrow q) \oplus (p \rightarrow \neg q)$
(c) $(p \vee q) \oplus (p \wedge q)$
(d) $(p \rightarrow q) \wedge (\neg p \rightarrow r)$

(e) $(\neg p \vee \neg q) \leftrightarrow (q \wedge r)$
(f) $(p \leftrightarrow q) \leftrightarrow (r \rightarrow s)$

Problem 2: De Morgan law (20 points).

(a) [10 points] Use a truth table to verify the first De Morgan law: $\neg(p \wedge q) \equiv \neg p \vee \neg q$.

(b) [10 points] Use De Morgan's laws to find the negation of each of the following statements:

1. Jan is rich and happy.
2. Carlos will bicycle or run tomorrow.
3. James is young and strong.
4. Rita will move to Oregon or Washington.

Problem 3: Equivalences (40 points).

(a) Determine whether $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology.
(b) Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent.
(c) Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.
(d) Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent.

Problem 4: Predicatives and Quantifiers (10 points) Let $Q(x, y)$ be the statement "$x + y = x - y$." If the domain for both variables consists of all integers, what are the truth values?

(a) $Q(1, 1)$
(b) $\forall y Q(1, y)$
(c) $\exists x Q(x, 2)$
(d) $\forall x \exists y Q(x, y)$
(e) $\exists y \forall x Q(x, y)$

[Optional Problem]:Logic Circuits (20 points)

(a) Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output $(p \wedge \neg r) \vee (\neg q \wedge r)$ from input bits p, q, and r.

(b) Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output $((\neg p \vee \neg r) \wedge \neg q) \vee (\neg p \wedge (q \vee r))$ from input bits p, q, and r.

[Hint]: Logic Circuit is one application of propositional logics. Please read "Logic Circuits" in Chapter 1.2 of our textbook from Page 41 to Page 43 before you start to solve this optional problem.