University at Albany, SUNY

College of Engineering and Applied Sciences, Computer Science

ICEN/ICSI-210: Discrete Structures

Spring 2019

Homework Set 12

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Assigned Date: Apr 29, 2019 (Monday).
Due Date: May 6, 2019 (Monday), 11:59 PM.
Collaboration Policy. Homeworks will be done individually: each student must hand in their own answers. Use of partial or entire solutions obtained from others or online is strictly prohibited.
Late Policy. If urgent or unusual circumstances prohibit you from submitting a homework assignment in time, please e-mail the instructor explaining the situation to get exempt from late penalty. Otherwise, any late submissions without consent from the instructor will result in exponential penalty – late for one day loses 25%, two days loses 50%, and so on and so forth. **Those submissions $\geq$ 3 hours after the deadline will be considered as "late submission" with no exemption.**
Submission Format. Electronic submission as a PDF file to blackboard is mandatory.

- You can write your solution in Word and save it as a PDF file.
- You also can write it on any physical papers and scan them to a PDF file.
- If you don't have condition to scan, you still can take pictures by your smart phone and convert images to a PDF file by the online tool (https://imagetopdf.com).
- If you have multiple PDF files, please combine them to a PDF file by the online tool (https://www.pdfmerge.com) or (https://www.ilovepdf.com/merge_pdf).

Problem 1: Properties of Relations (16 points) Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if

(a) $x \neq y$.
(b) $xy \geq 1$.
(c) $x = y + 1$ or $x = y - 1$.

(d) $x \equiv y \pmod 7$.
(e) x is a multiple of y.
(f) x and y are both negative or both nonnegative.
(g) $x = y^2$.
(h) $x \geq y^2$.

Problem 5: Relations (34 points)

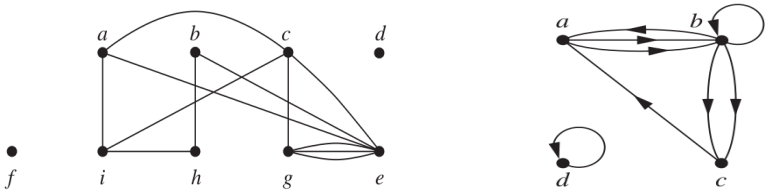
(a) [8 points] Let R be the relation $\{(1,2),(1,3),(2,3),(2,4),(3,1)\}$, and let S be the relation $\{(2,1),(3,1),(3,2),(4,2)\}$. Find $S \odot R$.

(b) [8 points] Represent the relation $R = \{(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)\}$ on the set $S = 1,2,3$ with a matrix (with the elements of this set listed in increasing order).

(c) [8 points] Represent the relation $R = \{(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)\}$ on the set $S = 1,2,3$ with a directed graph.

(d) [10 points] Let R be the relation on the set $\{1,2,3,4,5\}$ containing the ordered pairs $(1,3)$, $(2,4)$, $(3,1)$, $(3,5)$, $(4,3)$, $(5,1)$, $(5,2)$, and $(5,4)$. Find R^2, R^3, R^4, R^5, and R^6.

Problem 6: Graphs (30 points)



(a) A undirected graph. (b) A directed graph.

Figure 1: Two graphs.

(a) Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph in Figure 1a. Identify all isolated and pendant vertices.

(b) Determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph in Figure 1b.

(c) Draw these graphs: K_7 and $K_{4,4}$.

Problem 4: Trees (20 points)

(a) Answer these questions about the rooted tree illustrated in Figure 2: (1) Which vertex is the root? (2) Which vertices are internal? (3) Which vertices

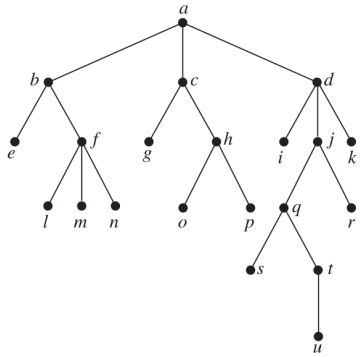


Figure 2: A rooted tree.

are leaves? (4) Which vertices are children of j ? (5) Which vertex is the parent of h? (6) Which vertices are siblings of o? (7) Which vertices are ancestors of m? and (8) Which vertices are descendants of b?

(b) Represent $(A \cap B) - (A \cup (B - A))$ using an ordered rooted tree.

[Optional] Extra Points (20 points)

Let R be a reflexive relation on a set A. Show that R^n is reflexive for all positive integers n. [**Hint**: *use mathmatic induction to prove.*]