University at Albany, SUNY

College of Engineering and Applied Sciences, Computer Science

ICEN/ICSI-210: Discrete Structures
Spring 2019

Homework Set 4

Chengjiang Long

Assigned Date: Feb 18, 2019 (Monday).

Due Date: Feb 25, 2018 (Monday), 11:59 PM.

Collaboration Policy. Homeworks will be done individually: each student must hand in their own answers. Use of partial or entire solutions obtained from others or online is strictly prohibited.

Late Policy. If urgent or unusual circumstances prohibit you from submitting a homework assignment in time, please e-mail the instructor explaining the situation to get exempt from late penalty. Otherwise, any late submissions without consent from the instructor will result in exponential penalty – late for one day loses 25%, two days loses 50%, and so on and so forth. **Those submissions $\geq$ 3 hours after the deadline will be considered as "late submission" with no exemption.**

Submission Format. Electronic submission as a PDF file to blackboard is mandatory.

- You can write your solution in Word and save it as a PDF file.

- You also can write it on any physical papers and scan them to a PDF file.

- If you don't have condition to scan, you still can take pictures by your smart phone and convert images to a PDF file by the online tool (https://imagetopdf.com).

- If you have multiple PDF files, please combine them to a PDF file by the online tool (https://www.pdfmerge.com) or (https://www.ilovepdf.com/merge_pdf).

Problem 1: Sequence Formulas or Rules (16 points) For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list.Assuming that your formula or rule is correct, determine the next three terms of the sequence.

(a) 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, 1, . . .

(b) 1, 2, 2, 3, 4, 4, 5, 6, 6, 7, 8, 8, . . .

(c) 1, 0, 2, 0, 4, 0, 8, 0, 16, 0, ...
(d) 3, 6, 12, 24, 48, 96, 192, ...
(e) 15, 8, 1, -6, -13, -20, -27, ...
(f) 3, 5, 8, 12, 17, 23, 30, 38, 47, ...
(g) 2, 16, 54, 128, 250, 432, 686, ...
(h) 2, 3, 7, 25, 121, 721, 5041, 40321, ...

Problem 2: Sequences and Summation (40 points) Compute each of these sums

(a) $\sum_{i=1}^{8} 3 \times 2^i$

(b) $\sum_{j=1}^{24} (-6j + 5)$

(c) $\sum_{k=1}^{519} \frac{1}{k(k+2)}$

(d) $\sum_{j=0}^{18} (j + (-1)^j)$

(e) $\sum_{j=0}^{18} (2 \cdot 3^j + 3 \cdot 2^j)$

(f) $\sum_{k=100}^{200} (2k + 3)$

(g) $\sum_{i=1}^{3} \sum_{j=1}^{2} (i - j)$

(h) $\sum_{i=0}^{2} \sum_{j=0}^{3} i^2 j^3$

Problem 3: Cardinality of Sets (18 points) Determine whether each of these sets is finite, countably infinite, or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.

(a) the integers greater than 10.
(b) the odd negative integers.
(c) the integers with absolute value less than 1,000,000.
(d) the real numbers between 0 and 2.
(e) the set $A \times \mathbf{Z}^+$ where $A = 2, 3$.
(f) the integers that are multiples of 10.

Problem 4: Matrices (28 points)
(a) [3 points] Find $\mathbf{A} + \mathbf{B}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 4 \\ -1 & 2 & 2 \\ 0 & -2 & -3 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} -1 & 3 & 5 \\ 2 & 2 & -3 \\ 2 & -3 & 0 \end{bmatrix}$$

(b) [5 points] Find $\mathbf{AB}$ with $\mathbf{A}$ and $\mathbf{B}$ given in (a).

(c) [5 points] Find the Boolean product of $\mathbf{A}$ and $\mathbf{B}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

(d) [15 points] Find $\mathbf{A}^{[2]}$, $\mathbf{A}^{[3]}$ and $\mathbf{A} \vee \mathbf{A}^{[2]} \vee \mathbf{A}^{[3]}$ with $\mathbf{A}$ given in (c).

[Optional] Extra Points (20 points)

(a) [2 points] Show that $\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0$, where $a_0, a_1, \ldots, a_n$ is a sequence of real numbers. This type of sum is called **telescoping**.

(b) [2 points] Use the identity $1/(k(k+1)) = 1/k - 1/(k+1)$ and (a) to compute $\sum_{k=1}^{n} 1/(k(k+1))$.

(c) [8 points] Sum both sides of the identity $k^2 - (k-1)^2 = 2k - 1$ from $k = 1$ to $k = n$ and use (a) to find **(i)** a formula for $\sum_{k=1}^{n}(2k-1)$ (the sum of the first n odd natural numbers), and **(ii)** a formula for $\sum_{k=1}^{n} k$.

(d) [8 points] Use the technique given in (a), together with the result of (c)[2], to derive the formula for $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$. [Hint: Take $a_k = k^3$ in the telescoping sum in (a).]