# University at Albany, SUNY

College of Engineering and Applied Sciences, Computer Science

# ICEN/ICSI-210: Discrete Structures

Spring 2019

# Homework Set 5

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**Assigned Date**: Feb 25, 2019 (Monday).
**Due Date**: Mar 4, 2019 (Monday), 11:59 PM.
**Collaboration Policy**. Homeworks will be done individually: each student must hand in their own answers. Use of partial or entire solutions obtained from others or online is strictly prohibited.
**Late Policy**. If urgent or unusual circumstances prohibit you from submitting a homework assignment in time, please e-mail the instructor explaining the situation to get exempt from late penalty. Otherwise, any late submissions without consent from the instructor will result in exponential penalty – late for one day loses 25%, two days loses 50%, and so on and so forth. **Those submissions $\geq$ 3 hours after the deadline will be considered as "late submission" with no exemption.**
**Submission Format**. Electronic submission as a PDF file to blackboard is mandatory.

- You can write your solution in Word and save it as a PDF file.
- You also can write it on any physical papers and scan them to a PDF file.
- If you don't have condition to scan, you still can take pictures by your smart phone and convert images to a PDF file by the online tool (https://imagetopdf.com).
- If you have multiple PDF files, please combine them to a PDF file by the online tool (https://www.pdfmerge.com) or (https://www.ilovepdf.com/merge_pdf).

**Problem 1: Algorithm (30 points)**

(a) Use the selection sort to sort 6, 2, 3, 1, 5, 4, showing the lists obtained at each step.

(b) Use the bubble sort to sort d, f, k, m, a, b, showing the lists obtained at each step.

(c) Use the greedy algorithm to make change using quarters, dimes, nickels, and pennies for 99 cents, showing the details at each step.

**Problem 2: The Growth of Function (30 points)**

(a) Show that $(x^2 + xy + xlogy)^3$ is $O(x^6y^3)$.

(b) Show that $x^5y^3 + x^4y^4 + x^3y^5$ is $\Omega(x^3y^3)$.

(c) Show that $3x^2 + x + 1$ is $\Theta(3x^2)$.

**Problem 3: The Growth of Multiple Functions (20 points)**

(a) Suppose that $f(x)$, $g(x)$, and $h(x)$ are functions such that $f(x)$ is $\Theta(g(x))$ and $g(x)$ is $\Theta(h(x))$. Show that $f(x)$ is $\Theta(h(x))$.

(b) Show that if $f_1(x)$ and $f_2(x)$ are functions from the set of positive integers to the set of real numbers and $f_1(x)$ is $\Theta(g_1(x))$ and $f_2(x)$ is $\Theta(g_2(x))$, then $(f_1 \circ f_2)(x)$ is $\Theta((g_1 \circ g_2)(x))$.

**Problem 4: Complexity (20 points)**

(a) Give a big-$O$ estimate for the number of operations, where an operation is a comparison or a multiplication, used in this segment of an algorithm (ignoring comparisons used to test the conditions in the **for** loops, where $a_1, a_2, \ldots, a_n$ are positive real numbers).

```
m := 0
for i := 1 to n
    for j := i + 1 to n
        m := max(a_i a_j, m)
```

(b) Show that the greedy algorithm for making change for $n$ cents using quarters, dimes, nickels, and pennies has $O(n)$ complexity measured in terms of comparisons needed.

**[Optional] Extra Points (20 points)**

The conventional algorithm for evaluating a polynomial $a_nx_n + a_{n-1}x_{n-1} + \ldots + a_1x + a_0$ at $x = c$ can be expressed in pseudocode by

```
procedure polynomial(c, a_0, a_1, ..., a_n : real numbers)
    power := 1
    y := a_0
    for i := 1 to n
        power := power × c
        y := y + a_i × power
    return y   {y = a_n c_n + a_{n-1} c_{n-1} + ... + a_1 c + a_0}
```

where the final value of y is the value of the polynomial at $x = c$.

(a) Evaluate $3x^2 + x + 1$ at $x = 2$ by working through each step of the algorithm showing the values assigned at each assignment step.

(b) Exactly how many multiplications and additions are used to evaluate a polynomial of degree n at $x = c$? (Do not count additions used to increment the loop variable.)