University at Albany, SUNY

College of Engineering and Applied Sciences, Computer Science

ICEN/ICSI-210: Discrete Structures

Spring 2019

Homework Set 9

Chengjiang Long

Assigned Date: Apr 1, 2019 (Monday).
Due Date: Apr 8, 2019 (Monday), 11:59 PM.
Collaboration Policy. Homeworks will be done individually: each student must hand in their own answers. Use of partial or entire solutions obtained from others or online is strictly prohibited.
Late Policy. If urgent or unusual circumstances prohibit you from submitting a homework assignment in time, please e-mail the instructor explaining the situation to get exempt from late penalty. Otherwise, any late submissions without consent from the instructor will result in exponential penalty – late for one day loses 25%, two days loses 50%, and so on and so forth. **Those submissions $\geq$ 3 hours after the deadline will be considered as "late submission" with no exemption.**
Submission Format. Electronic submission as a PDF file to blackboard is mandatory.

- You can write your solution in Word and save it as a PDF file.
- You also can write it on any physical papers and scan them to a PDF file.
- If you don't have condition to scan, you still can take pictures by your smart phone and convert images to a PDF file by the online tool (https://imagetopdf.com).
- If you have multiple PDF files, please combine them to a PDF file by the online tool (https://www.pdfmerge.com) or (https://www.ilovepdf.com/merge_pdf).

Problem 1: Recursive Definition (20 points) Give a recursive definition of:

(a) the set of odd positive integers.
(b) the set of even positive integers.
(c) the set of positive integer powers of 3.
(d) the set of positive integers congruent to 2 modulo 3.

(e) the set of positive integers that are multiples of 5.

Problem 2: Strong Induction and Structural Induction (30 points) Let S be the subset of the set of ordered pairs of integers defined recursively by

Basis step: $(0, 0) \in S$.

Recursive step: If $(a, b) \in S$, then $(a + 2, b + 3) \in S$ and $(a + 3, b + 2) \in S$.

(a) List the elements of S produced by the first five applications of the recursive definition.

(b) Show that $5|a + b$ when $(a, b) \in S$ [**Hint**: you can use strong induction or structural induction].

Problem 3: Recursive Algorithms (30 points)

(a) [8 points] Give a recursive algorithm for finding the sum of the first n odd positive integers.

(b) [10 points] Give a recursive algorithm for finding $n!$ **mod** m whenever n and m are positive integers.

(c) [12 points] Devise a recursive algorithm for computing n^2 where n is a nonnegative integer, using the fact that $(n + 1)^2 = n^2 + 2n + 1$. Then prove that this algorithm is correct.

Problem 4: Basic Counting Rules (20 points)

(a) How many positive integers between 50 and 100 are divisible by 11?

(b) How many positive integers less than 1000 are divisible by exactly one of 7 and 11?

(c) How many bit strings of length seven either begin with two 0s or end with three 1s?

(d) How many ways are there to arrange the letters a, b, c, and d such that a is not followed immediately by b?

[Optional] Extra Points (20 points)

(a) [5 points] Six different airlines fly from New York to Denver and seven fly from Denver to San Francisco. How many different pairs of airlines can you choose on which to book a trip from NewYork to San Francisco via Denver, when you pick an airline for the flight to Denver and an airline for the continuation flight to San Francisco?

(b) [5 points] Suppose that at some future time every telephone in the world is assigned a number that contains a country code 1 to 3 digits long, that is, of the form X, XX, or XXX, followed by a 10-digit telephone number of the form NXX-NXX-XXXX (as described in Example 8). How many different telephone numbers would be available worldwide under this numbering plan?

(c) [5 points] A key in theVigenère cryptosystem is a string of English letters, where the case of the letters does not matter. How many different keys for this cryptosystem are there with three, four, five, or six letters?

(d) [5 points] A wired equivalent privacy (WEP) key for a wireless fidelity (WiFi) network is a string of either 10, 26, or 58 hexadecimal digits. How many different WEP keys are there?