University at Albany, SUNY

College of Engineering and Applied Sciences, Computer Science

ISEN/ISCI-210: Discrete Structures

Fall 2018

Final Exam

Name: ________________ ID #: ________________ Score: ________________

- This is a OPEN BOOK & OPEN NOTE exam. Also, you cannot access the Internet or use your laptop computer. Do the exam independently.
- There are a total of 100 points in the exam. Plan your work accordingly.
- Write out the steps for all problems to receive the full credit. Use additional pages if necessary.
- Date: Dec 17th, 2018.
- Location: Lecture center hall 25.
- Time: 3:30 pm - 5:30 pm (can be extended to 6:30 pm).

Problem	Points	Scores
Problem 1: True or False	10	
Problem 2: Logic and Proofs	8	
Problem 3: Functions, Sequences and Sums	15	
Problem 4: Algorithm Complexity	7	
Problem 5: Integer Representations and Modular Arithmetic	15	
Problem 6: Induction and Recursion	10	
Problem 7: Counting and Discrete Probability	15	
Problem 8: Relations	20	

Problem 1: True or False (10 points)

(1) $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$ are logically equivalent.
○ True ○ False

(2) Let $f : Z \to Z$ be defined by $f(x) = x^2 - 5x + 5$. Then f is a onto (surjective) function.
○ True ○ False

(3) The sequence $1, 1, 1, \ldots, 1$ is an arithmetic and geometric sequence.
○ True ○ False

(4) For algorithm complexity, $O(1) < O(logn) < O(n) < O(nlogn) < O(n^2) < O(n^3) < O(2^n) < O(n!)$.
○ True ○ False

(5) Turing's code, either version 1.0 or version 2.0, cannot be decrypted by a third-part adversary.
○ True ○ False

(6) Strong induction uses the basic step $P(1)$ and the inductive step $P(n-1) \to P(n)$.
○ True ○ False

(7) The sum rule can be used in only two disjoint sets, while the inclusion-exclusion principle can be used in any cases.
○ True ○ False

(8) Let R be the relation $\{(a, b)|a \equiv b(\text{mod } 7)\}$. Then the equivalence class of 5 is equal to the equivalence class of 19, *i.e.*, $[5] = [19]$.
○ True ○ False

(9) Let $G = (V, E)$ be an undirected graph with e edges (*i.e.*, $|E| = e$). Then $\sum_{v \in V} \deg(v) = e$.
○ True ○ False

(10) Given a connected graph $G = (V, E)$, remove an edge from any circuit and then we can get at least one spanning tree.
○ True ○ False

Problem 2: Logic and Proofs (8 points)

(1) [**3 points**] Describe a way to prove the biconditional $p \leftrightarrow q$.

(2) [**5 points**] Prove the statement: "The integer $3n + 2$ is odd if and only if the integer $9n + 5$ is even, where n is an integer.".

Problem 3: Functions, Sequences and Sums (15 points)

(1) [**5 points**] Show that $f(x) = 2x^2 + 9x + 99$ is a one-to-one function from **Z** to **Z**.

(2) [**5 points**] Compute the sum $\sum_{j=1}^{30} (2^{j+1} - 2^j)$.

(3) [**5 points**] Compute the sum $\sum_{k=1}^{30} \frac{1}{k(k+2)}$.

Problem 4: Algorithm Complexity (7 points)

(1) **[3 points]** State the definition of the fact that $f(n)$ is $O(g(n))$, where $f(n)$ and $g(n)$ are functions from the set of positive integers to the set of real numbersr.

(2) **[4 points]** Use the definition of the fact that $f(n)$ is $O(g(n))$ to show that $(logn + n^2)^3$ is $O(n^6)$.

Problem 5: Integer Representations and Modular Arithmetic (15 points)

(1) [**4 points**] Convert $(1101100101011011)_2$ to octal and hexadecimal representations.

(2) [**4 points**] Convert $(7206)_8$ and $(A0EB)_{16}$ to a binary representation.

(3) [**7 points**] Use Modular Exponention Algorithm to find 11^{43} mod 9.

Problem 6: Induction and Recursion (10 points)

(1) [**7 points**] Use mathematical induction to show that $1 \cdot 2^0 + 2 \cdot 2^1 + 3 \cdot 2^2 + \ldots + n \cdot 2^{n-1} = (n-1) \cdot 2^n + 1$ whenever n is a positive integer.

(2) [**3 points**] Using your own words to explicitly interpret a set S which is a recursively defined by the following two rules:

- Rule 1: $2 \in S$.
- Rule 2: $x \in S, y \in S \rightarrow x + y \in S$.

Problem 7: Counting and Discrete Probability (15 points)

(1) [**5 points**] We say a bit string is unbalanced if there are more ones than zeroes or more zeros than ones. How many 10-bit strings are unbalanced?

(2) [**5 points**] Use binomial theorem to show that if n is an integer then

$$\sum_{k=0}^{n} 3^k \binom{n}{k} = 4^n \tag{1}$$

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(3) [**5 points**] Use Bayes rule to find $p(F|E)$ if $p(E|F) = 1/3$, $p(E|\bar{F}) = 1/4$, $p(F) = 2/3$, and $p(\bar{F}) = 1/3$, where E and F are events from a sample space S.

Problem 8: Relations (20 points)

(1) **[10 points]** Show that the relation $R = \{(x, y) | x \equiv y (\bmod 5)\}$ is reflexive, symmetric, and transitive.

(2) **[10 points]** Let R_1 and R_2 be relations on a set $A = \{a, b, c, d\}$ represented by the following directed graphs. Find the relations $R_1 \cap R_2$, $R_1 \cup R_2$, $R_1 \circ R_2$, and R_2^2. [Hint: *You convert the represention of both R_1 and R_2 to zero-one matrices.*]

(a) R_1. (b) R_2.

Figure 1: The directed graphs for two relations R_1 and R_2.