

University at Albany, SUNY

College of Engineering and Applied Sciences, Computer Science

ISEN/ISCI-210: Discrete Structures

Fall 2018

Final Exam

Name: _____ ID #: _____ Score: _____

- This is a OPEN BOOK & OPEN NOTE exam. Also, you cannot access the Internet or use your laptop computer. Do the exam independently.
- There are a total of 100 points in the exam. Plan your work accordingly.
- Write out the steps for all problems to receive the full credit. Use additional pages if necessary.
- Date: Dec 17th, 2018.
- Location: Lecture center hall 25.
- Time: 3:30 pm - 5:30 pm (can be extended to 6:30 pm).

Problem	Points	Scores
Problem 1: True or False	10	
Problem 2: Logic and Proofs	8	
Problem 3: Functions, Sequences and Sums	15	
Problem 4: Algorithm Complexity	7	
Problem 5: Integer Representations and Modular Arithmetic	15	
Problem 6: Induction and Recursion	10	
Problem 7: Counting and Discrete Probability	15	
Problem 8: Relations	20	

Problem 1: True or False (10 points)

- (1) $(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$ are logically equivalent.
 True False
- (2) Let $f : Z \rightarrow Z$ be defined by $f(x) = x^2 - 5x + 5$. Then f is a onto (surjective) function.
 True False
- (3) The sequence $1, 1, 1, \dots, 1$ is an arithmetic and geometric sequence.
 True False
- (4) For algorithm complexity, $O(1) < O(\log n) < O(n) < O(n \log n) < O(n^2) < O(n^3) < O(2^n) < O(n!)$.
 True False
- (5) Turing's code, either version 1.0 or version 2.0, cannot be decrypted by a third-part adversary.
 True False
- (6) Strong induction uses the basic step $P(1)$ and the inductive step $P(n - 1) \rightarrow P(n)$.
 True False
- (7) The sum rule can be used in only two disjoint sets, while the inclusion-exclusion principle can be used in any cases.
 True False
- (8) Let R be the relation $\{(a, b) | a \equiv b \pmod{7}\}$. Then the equivalence class of 5 is equal to the equivalence class of 19, *i.e.*, $[5] = [19]$.
 True False
- (9) Let $G = (V, E)$ be an undirected graph with e edges (*i.e.*, $|E| = e$). Then $\sum_{v \in V} \deg(v) = e$.
 True False
- (10) Given a connected graph $G = (V, E)$, remove an edge from any circuit and then we can get at least one spanning tree.
 True False

Problem 2: Logic and Proofs (8 points)

- (1) [**3 points**] Describe a way to prove the biconditional $p \leftrightarrow q$.
- (2) [**5 points**] Prove the statement: “The integer $3n + 2$ is odd if and only if the integer $9n + 5$ is even, where n is an integer.”.

Problem 3: Functions, Sequences and Sums (15 points)

(1) [5 points] Show that $f(x) = 2x^2 + 9x + 99$ is a one-to-one function from \mathbf{Z} to \mathbf{Z} .

(2) [5 points] Compute the sum $\sum_{j=1}^{30} (2^{j+1} - 2^j)$.

(3) [5 points] Compute the sum $\sum_{k=1}^{30} \frac{1}{k(k+2)}$.

Problem 4: Algorithm Complexity (7 points)

(1) [**3 points**] State the definition of the fact that $f(n)$ is $O(g(n))$, where $f(n)$ and $g(n)$ are functions from the set of positive integers to the set of real numbers.

(2) [**4 points**] Use the definition of the fact that $f(n)$ is $O(g(n))$ to show that $(\log n + n^2)^3$ is $O(n^6)$.

Problem 5: Integer Representations and Modular Arithmetic (15 points)

- (1) [4 points] Convert $(1101100101011011)_2$ to octal and hexadecimal representations.
- (2) [4 points] Convert $(7206)_8$ and $(A0EB)_{16}$ to a binary representation.
- (3) [7 points] Use Modular Exponentiation Algorithm to find $11^{43} \bmod 9$.

Problem 6: Induction and Recursion (10 points)

(1) [7 points] Use mathematical induction to show that $1 \cdot 2^0 + 2 \cdot 2^1 + 3 \cdot 2^2 + \dots + n \cdot 2^{n-1} = (n-1) \cdot 2^n + 1$ whenever n is a positive integer.

(2) [3 points] Using your own words to explicitly interpret a set S which is a recursively defined by the following two rules:

- Rule 1: $2 \in S$.
- Rule 2: $x \in S, y \in S \rightarrow x + y \in S$.

Problem 7: Counting and Discrete Probability (15 points)

(1) [5 points] We say a bit string is unbalanced if there are more ones than zeroes or more zeros than ones. How many 10-bit strings are unbalanced?

(2) [5 points] Use binomial theorem to show that if n is an integer then

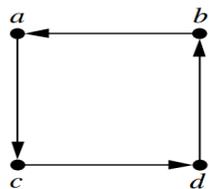
$$\sum_{k=0}^n 3^k \binom{n}{k} = 4^n \quad (1)$$

(3) [5 points] Use Bayes rule to find $p(F|E)$ if $p(E|F) = 1/3$, $p(E|\bar{F}) = 1/4$, $p(F) = 2/3$, and $p(\bar{F}) = 1/3$, where E and F are events from a sample space S .

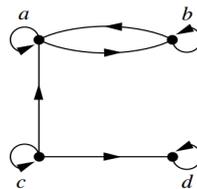
Problem 8: Relations (20 points)

(1) [10 points] Show that the relation $R = \{(x, y) | x \equiv y \pmod{5}\}$ is reflexive, symmetric, and transitive.

(2) [10 points] Let R_1 and R_2 be relations on a set $A = \{a, b, c, d\}$ represented by the following directed graphs. Find the relations $R_1 \cap R_2$, $R_1 \cup R_2$, $R_1 \circ R_2$, and R_2^2 . [Hint: You convert the representation of both R_1 and R_2 to zero-one matrices.]



(a) R_1 .



(b) R_2 .

Figure 1: The directed graphs for two relations R_1 and R_2 .