University at Albany, SUNY

College of Engineering and Applied Sciences, Computer Science

ISEN/ISCI-210: Discrete Structures

Fall 2018

Homework Set 5

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Assigned Date: Nov 28, 2018 (Wednesday).
Due Date: Dec 7, 2018 (Friday).
Collaboration Policy. Homeworks will be done individually: each student must hand in their own answers. Use of partial or entire solutions obtained from others or online is strictly prohibited.
Late Policy. No late submissions will be allowed without consent from the instructor. If urgent or unusual circumstances prohibit you from submitting a homework assignment in time, please e-mail me explaining the situation. ***Those submissions $\geq$ 12 hours after the deadline will be considered as "late submission".***
Submission Format. Electronic submission of a PDF file is mandatory.

Problem 1: Discrete Probabilities (20 points)

(a) [3 points] What is the probability that a positive integer not exceeding 100 selected at random is divisible by 3?

(b) [4 points] What is the probability that a positive integer not exceeding 100 selected at random is divisible by 5 or 7?

(c) [5 points] Suppose that E and F are events such that $p(E) = 0.7$ and $p(F) = 0.5$. Show that $p(E \cup F) \geq 0.7$ and $p(E \cap F) \geq 0.2$.

(d) [3 points] Suppose that E and F are events in a sample space and $p(E) = 2/3$, $p(F) = 3/4$, and $p(F|E) = 5/8$. Find $p(E|F)$.

(e) [5 points] Suppose that E, F_1, F_2, and F_3 are events from a sample space S and that F_1, F_2, and F_3 are pairwise disjoint and their union is S. Find $p(F_2|E)$ if $p(E|F_1) = 2/7$, $p(E|F_2) = 3/8$, $p(E|F_3) = 1/2$, $p(F_1) = 1/6$, $p(F_2) = 1/2$, and $p(F_3) = 1/3$.

Problem 2: Bayes Rule (10 points) Suppose that 8% of the patients tested in a clinic are infected with HIV. Furthermore, suppose that when a blood test for HIV is given, 98% of the patients infected with HIV test positive and that

3% of the patients not infected with HIV test positive. What is the probability that

(a) a patient testing positive for HIV with this test is infected with it?
(b) a patient testing positive for HIV with this test is not infected with it?
(c) a patient testing negative for HIV with this test is infected with it?
(d) a patient testing negative for HIV with this test is not infected with it?

Problem 3: Expected Vaules and Variance (16 points)

(a) [4 points] What is the expected number of heads that come up when a fair coin is flipped five times?

(b) [4 points] What is the expected number of times a 6 appears when a fair die is rolled 10 times?

(c) [8 points] Let X be the number appearing on the first die when two fair dice are rolled and let Y be the sum of the numbers appearing on the two dice. Show that $E(X)E(Y) \neq E(XY)$.

Problem 4: Properties of Relations (16 points) Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if

(a) $x \neq y$.
(b) $xy \geq 1$.
(c) $x = y + 1$ or $x = y - 1$.
(d) $x \equiv y \pmod 7$.
(e) x is a multiple of y.
(f) x and y are both negative or both nonnegative.
(g) $x = y^2$.
(h) $x \geq y^2$.

Problem 5: Relations (20 points)

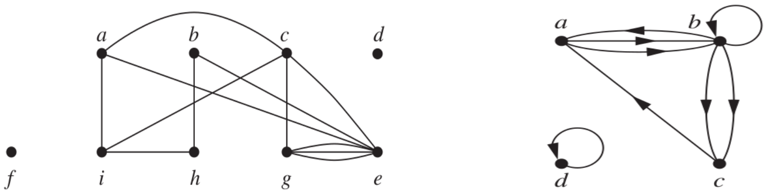
(a) [4 points] Let R be the relation $\{(1,2),(1,3),(2,3),(2,4),(3,1)\}$, and let S be the relation $\{(2,1),(3,1),(3,2),(4,2)\}$. Find $S \odot R$.

(b) [3 points] Represent the relation $R = \{(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)\}$ on the set $S = 1, 2, 3$ with a matrix (with the elements of this set listed in increasing order).

(c) [3 points] Represent the relation $R = \{(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)\}$ on the set $S = 1, 2, 3$ with a directed graph.

(d) [10 points] Let R be the relation on the set $\{1, 2, 3, 4, 5\}$ containing the ordered pairs $(1,3)$, $(2,4)$, $(3,1)$, $(3,5)$, $(4,3)$, $(5,1)$, $(5,2)$, and $(5,4)$. Find R^2, R^3, R^4, R^5, and R^6.

Problem 6: Graphs and Trees (18 points)



(a) A undirected graph. (b) A directed graph.

Figure 1: Two graphs.

(a) [3 points] Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph in Figure 1a. Identify all isolated and pendant vertices.

(b) [3 points] Determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph in Figure 1b.

(c) [4 points] Draw these graphs: K_7 and $K_{4,4}$.

(d) [4 points] Answer these questions about the rooted tree illustrated in Figure 2: (1) Which vertex is the root? (2) Which vertices are internal? (3) Which vertices are leaves? (4) Which vertices are children of j ? (5) Which vertex is the parent of h? (6) Which vertices are siblings of o? (7) Which vertices are ancestors of m? and (8) Which vertices are descendants of b?

(e) [4 points] Represent $(A \cap B) - (A \cup (B - A))$ using an ordered rooted tree.

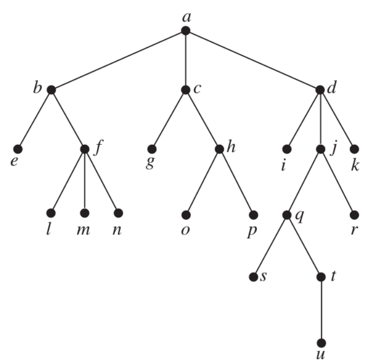


Figure 2: A rooted tree.

[Optional] Extra Points (20 points)

(a) [6 points] What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up heads?

(b) [6 points] Use mathematical induction to prove that if E_1 , E_2, …, E_n is a sequence of n pairwise disjoint events in a smaple space S, where n is a positive integer, then $p(\bigcup_{i=1}^{n} E_i) = \sum_{i=1}^{n} p(E_i)$.

(c) [8 points] Let R be a reflexive relation on a set A. Show that R^n is reflexive for all positive integers n. [**Hint**: *use mathmatic induction to prove.*]